METIS*

A Software Package for Partitioning Unstructured Graphs, Partitioning Meshes, and Computing Fill-Reducing Orderings of Sparse Matrices

Version 4.0

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Metis [MEE tis]: *'Metis' is the Greek word for wisdom. Metis was a titaness in Greek mythology. She was the consort of Zeus and the mother of Athena. She presided over all wisdom and knowledge.*

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Contents

1 Introduction

Algorithms that find a good partitioning of highly unstructured graphs are critical for developing efficient solutions for a wide range of problems in many application areas on both serial and parallel computers. For example, large-scale numerical simulations on parallel computers, such as those based on finite element methods, require the distribution of the finite element mesh to the processors. This distribution must be done so that the number of elements assigned to each processor is the same, and the number of adjacent elements assigned to different processors is minimized. The goal of the first condition is to balance the computations among the processors. The goal of the second condition is to minimize the communication resulting from the placement of adjacent elements to different processors. Graph partitioning can be used to successfully satisfy these conditions by first modeling the finite element mesh by a graph, and then partitioning it into equal parts.

Graph partitioning algorithms are also used to compute fill-reducing orderings of sparse matrices. These fill-reducing orderings are useful when direct methods are used to solve sparse systems of linear equations. A good ordering of a sparse matrix dramatically reduces both the amount of memory as well as the time required to solve the system of equations. Furthermore, the fill-reducing orderings produced by graph partitioning algorithms are particularly suited for parallel direct factorization as they lead to high degree of concurrency during the factorization phase.

Graph partitioning is also used for solving optimization problems arising in numerous areas such as design of very large scale integrated circuits (VLSI), storing and accessing spatial databases on disks, transportation management, and data mining.

2 What is METIS

METIS is a software package for partitioning large irregular graphs, partitioning large meshes, and computing fill-reducing orderings of sparse matrices. The algorithms in METIS are based on multilevel graph partitioning described in [8, 7, 6]. Traditional graph partitioning algorithms compute a partition of a graph by operating directly on the original graph as illustrated in Figure 1(a). These algorithms are often too slow and/or produce poor quality partitions.

Multilevel partitioning algorithms, on the other hand, take a completely different approach [5, 8, 7]. These algorithms, as illustrated in Figure 1(b), reduce the size of the graph by collapsing vertices and edges, partition the smaller graph, and then uncoarsen it to construct a partition for the original graph. METIS uses novel approaches to successively reduce the size of the graph as well as to further refine the partition during the uncoarsening phase. During coarsening, METIS employs algorithms that make it easier to find a high-quality partition at the coarsest graph. During refinement, METIS focuses primarily on the portion of the graph that is close to the partition boundary. These highly tuned algorithms allow METIS to quickly produce high-quality partitions for a large variety of graphs.

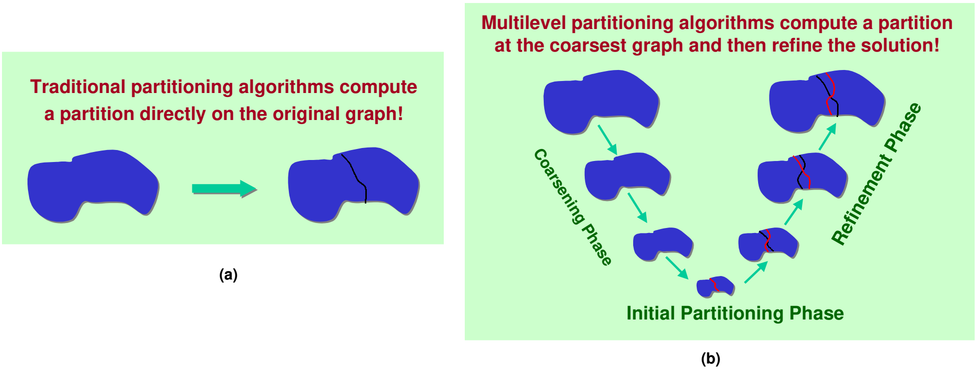


Figure 1: (a) Traditional partitioning algorithms. (b) Multilevel partitioning algorithms.

The advantages of METIS compared to other similar packages are the following:

☞ **Provides high quality partitions!**
Experiments on a large number of graphs arising in various domains including finite element methods, linear programming, VLSI, and transportation show that METIS produces partitions that are consistently better than those produced by other widely used algorithms. The partitions produced by METIS are consistently 10% to 50% better than those produced by spectral partitioning algorithms [1, 4].

☞ **It is extremely fast!**
Experiments on a wide range of graphs has shown that METIS is one to two orders of magnitude faster than other widely used partitioning algorithms. Figure 2 shows the amount of time required to partition a variety of graphs in 256 parts for two different architectures, an R10000-based SGI Challenge and a Pentium Pro-based personal computer. Graphs containing up to four million vertices can be partitioned in 256 parts in well under a minute on today's scientific workstations. The run time of METIS is comparable to (or even smaller than) the run time of some geometric partitioning algorithms that often produce much worse partitions.

☞ **Provides low fill orderings!**
The fill-reducing orderings produced by METIS are substantially better than those produced by other widely used algorithms including multiple minimum degree. For many classes of problems arising in scientific computations and linear programming, METIS is able to reduce the storage and computational requirements of sparse matrix factorization methods by up to an order of magnitude. Moreover, unlike multiple minimum degree, the elimination trees produced by METIS are suited for parallel direct factorization. Furthermore, as Figure 2 illustrates, METIS is able to compute these ordering very fast. Matrices with over two hundred thousand rows can be reordered in just a few seconds on current generation workstations and PCs.

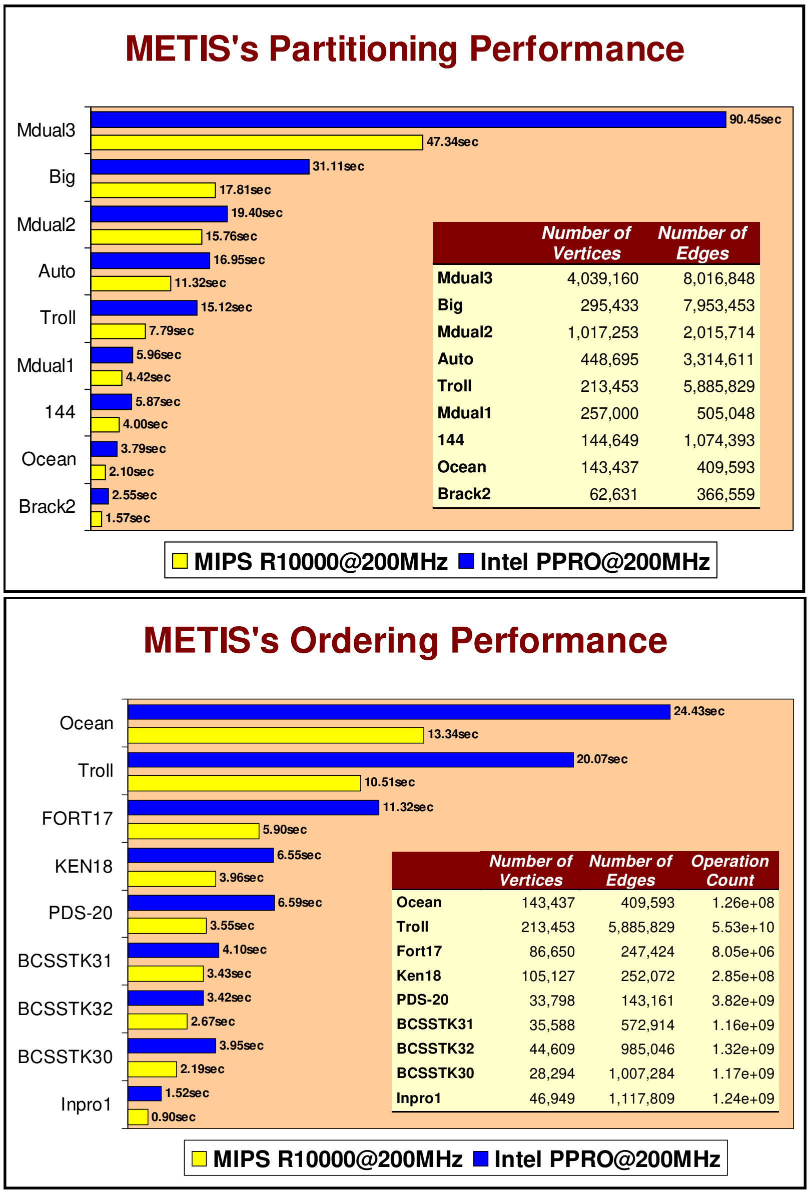


Figure 2: The amount of time required by METIS to partition various graphs in 256 parts and the amount of time required by METIS to compute fill-reducing orderings for various sparse matrices.

The rest of this manual is organized as follows: Section 4 describes the user interface to the stand-alone programs provided by METIS. Section 5 describes the stand-alone library that implements the various algorithms implemented in METIS. Finally, Section 6 describes the system requirements for the METIS package.

3 What is New in This Version

The latest version of METIS contains a number of changes over the previous major release (version 3.0). Most of these changes are concentrated on the graph and mesh partitioning routines and they marginally affect the sparse matrix re-ordering routines. Table 1 describes which programs and routines of METISlib have been changed and the new routines in METISlib. In the rest of this section we briefly describe some of the major changes.

Multi-Constraint Partitioning METIS now includes partitioning routines that can be used to partition a graph in the presence of multiple balancing constraints. The idea is that each vertex has a vector of weights of size m associated with it, and the objective of the partitioning algorithm is to minimize the edgecut subject to the constraints that each one of the m weights is equally distributed among the domains. For example, if the first weight corresponds to the amount of computation and the second weight corresponds to the amount of storage required for each element, then the partitioning computed by the new algorithms will balance both the computation performed in each domain as well as the amount of memory that it requires. Also, multi-phase (multi-physics) computations can use the new partitioning algorithm to simultaneously balance the computations performed in each phase. The multi-constraint partitioning algorithms and their applications are further described in [6].

The multi-constraint partitioning algorithm is implemented by the METIS_mCPartGraphRecursive and METIS_mCPartGraphKway routines that are based on the multilevel recursive bisection and the multilevel k-way partitioning paradigms, respectively. Also, the pmetis and the kmetis programs have been overloaded to invoke the multi-constraint partitioner when the input graph contains multiple vertex weights (Section 4.5.1 describes how the format of the input graph file has been extended to allow you to specify multiple vertex weights).

Minimizing the Total Communication Volume The objective of the traditional graph partitioning problem is to compute a balanced k-way partitioning such that the number of edges (or in the case of weighted graphs the sum of their weights) that straddle different partitions is minimized. When partitioning is used to distribute a graph or a mesh among the processors of a parallel computer, the objective of minimizing the edgecut is only an approximation of the true communication cost resulting from the partitioning. Despite that, for a wide range of problems, by minimizing the edgecut, the partitioning algorithms also minimize the communication cost reasonably well.

However, there are cases in which a partitioning algorithm can significantly reduce the communication cost by directly minimizing this objective (as opposed to the edgecut). METIS now provides the METIS_PartGraphVKway and METIS_WPartGraphVKway routines that directly minimize the communication cost as defined by the total communication volume resulted by the partitioning (see Section 5.3 for a precise definition of this objective function). Note that for these routines to provide meaningful partitionings, the connectivity of the graph should reflect the true information exchange requirements of the underlying computation.

Minimizing the Maximum Connectivity of the Subdomains The communication cost resulting from a k-way partitioning in general depends on the following factors: (i) the total communication volume, (ii) the maximum amount of data that any particular processor needs to send and receive; and (iii) the number of messages a processor needs to send and receive. The partitioning routines in earlier versions of METIS concentrated only on the first factor (by minimizing the edgecut). In this release, METIS also provides support for minimizing the third factor (which essentially reduces the number of startups) and indirectly (up to a point) reduces the second factor. Experiments have shown that for most graphs corresponding to finite element meshes, the new release of METIS is able to reduce the maximum (and total) number of adjacent subdomains considerably—especially when the graph is partitioned in a relatively large number of partitions (*e.g.*, greater than 30). For most 3D finite elements graphs, the maximum number of subdomains for a 50-way partition has been reduced from around 25 to around 16.

This enhancement is provided as a refinement option for both the METIS_PartGraphKway and METIS_PartGraphVKway routines, and is the default option of kmetis and METIS_PartGraphKway.

Reducing the Number of Non-Contiguous Subdomains A k-way partitioning of a contiguous graph can often lead to some subdomains being assigned non-contiguous portions of the graph. For many problems, the non-

Changes in METIS's stand-alone programs	
pmetis	It has been over-loaded to invoke the multi-constraint partitioning algorithm when the graph contains multiple vertex weights.
kmetis	It has been over-loaded to invoke the multi-constraint partitioning algorithm when the graph contains multiple vertex weights. The partitioning algorithm has been modified to also minimize the connectivity of the subdomains. A pre- and post-refinement step is applied that tries to reduce the number of non-contiguous subdomains.
partnmesh partdmesh	The partitioning algorithm has been modified to also minimize the connectivity of the subdomains.

Changes in METISlib's routines	
METIS_PartGraphKway METIS_WPartGraphKway	A new refinement algorithm has been added that also minimizes the connectivity of the subdomains. This new algorithm has been made the default option. A pre- and post-refinement step is applied that tries to reduce the number of non-contiguous subdomains.
METIS_PartGraphVKway METIS_WPartGraphVKway	This is a new set of routines that compute a k-way partitioning whose objective is to minimize the total communication volume.
METIS_mCPartGraphRecursive METIS_mCPartGraphKway	This is a new set of routines that compute a k-way partitioning subject to multiple balancing constraints.

Table 1: Summary of the changes in METIS and METISlib.

contiguity is a result of the underlying geometry and often leads to better quality partitions. Nevertheless, there are cases in which the partitioning algorithm is fooled and breaks certain domains. METIS now provides support for eliminating such spurious non-contiguous subdomains.

This support is provided as a default option for both the METIS_PartGraphKway and METIS_PartGraphVKway routines, and the kmetis program.

4 METIS's Stand-Alone Programs

METIS provides a variety of programs that can be used to partition graphs, partition meshes, compute fill-reducing orderings of sparse matrices, as well as programs to convert meshes into graphs appropriate for METIS's graph partitioning programs.

The rest of this section provides detailed descriptions about the functionality of these programs, how to use them, the format of the input files required by them, and the format of the produced output files.

4.1 Graph Partitioning Programs

METIS provides two programs `pmetis` and `kmetis` for partitioning an unstructured graph into k equal size parts. The partitioning algorithm used by `pmetis` is based on multilevel recursive bisection described in [8], whereas the partitioning algorithm used by `kmetis` is based on multilevel k-way partitioning described in [7]. Both of these programs are able to produce high quality partitions. However, depending on the application, one program may be preferable than the other. In general, `kmetis` is preferred when it is necessary to partition graphs into more than eight partitions. For such cases, `kmetis` is considerably faster than `pmetis`. On the other hand, `pmetis` is preferable for partitioning a graph into a small number of partitions.

Both `pmetis` and `kmetis` are invoked by providing two arguments at the command line as follows:

pmetis *GraphFile* *Nparts*
kmetis *GraphFile* *Nparts*

The first argument *GraphFile*, is the name of the file that stores the graph (whose format is described in Section 4.5.1), while the second argument *Nparts*, is the number of partitions that is desired. Both `pmetis` and `kmetis` can partition a graph into an arbitrary number of partitions. Upon successful execution, both programs display statistics regarding the quality of the computed partitioning and the amount of time taken to perform the partitioning. The actual partitioning is stored in a file named ***GraphFile.part.Nparts***, whose format is described in Section 4.6.1.

Figure 3 shows the output of `pmetis` and `kmetis` for partitioning a graph into 100 parts. From this figure we see that both programs initially print information about the graph, such as its name, the number of vertices (*#Vertices*), the number of edges (*#Edges*), and also the number of desired partitions (*#Parts*). Next, they print some information regarding the quality of the partitioning. Specifically, they report the number of edges being cut (*Edge-Cut*) by the partitioning, as well as the balance of the partitioning[1]. Finally, both `pmetis` and `kmetis` show the time taken by the various phases of the algorithm. All times are in seconds. For this particular example, `pmetis` required a total of 17.070 seconds, of which 13.850 seconds was taken by the partitioning algorithm itself, and the rest was to read the graph itself. Similarly, `kmetis` required a total of 6.790 seconds, of which 3.570 seconds was taken by the partitioning algorithm itself. As you can see from this example, `kmetis` is considerably faster than `pmetis`, and it produces a partitioning that is slightly better than that produced by `pmetis`.

Figure 4 shows the output of `pmetis` and `kmetis` for partitioning a graph into 16 parts subject to three balancing constraints. Both `pmetis` and `kmetis` have been *over-loaded* to invoke the multi-constraint partitioning routines whenever the input graph file specifies more that one set of vertex weights. Comparing the output of Figure 4 to that of Figure 3 we see that when `pmetis` and `kmetis` operate in the multi-constraint mode they display some additional information regarding the number of constraints and also the balance of the computed partitioning with respect to each one of these constraints. In this example, `pmetis` was able to balance the three constraints within 1%, 3%, and 2%, respectively. Note that for multi-constraint partitioning, for small number of partitions `pmetis` outperforms `kmetis` in terms of partitioning quality. However, for larger number of partitions `kmetis` achieves better quality and is more robust in simultaneously balancing the various constraints.

[1] For a k way partition of a graph with n vertices, let m be the size of the largest part produced by the k-way partitioning algorithm. The balance of the partitioning is defined as km/n, and is essentially the load imbalance induced by non-equal partitions. `pmetis` produces partitions that are perfectly balanced at each bisection level, however, some small load imbalance may result due to the $\log k$ levels of recursive bisection. In general, the load imbalance is less than 1%. `kmetis` produces partitions that are not perfectly balanced, but the algorithm limits the load imbalance to 3%.

```
prompt% pmetis brack2.graph 100
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: brack2.graph, #Vertices: 62631, #Edges: 366559, #Parts: 100

Recursive Partitioning... -------------------------------------------
  100-way Edge-Cut:   37494, Balance:  1.00

Timing Information --------------------------------------------------
  I/O:                          0.820
  Partitioning:                 6.110   (PMETIS time)
  Total:                        6.940
**********************************************************************

prompt% kmetis brack2.graph 100
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: brack2.graph, #Vertices: 62631, #Edges: 366559, #Parts: 100

K-way Partitioning... -----------------------------------------------
  100-way Edge-Cut:   37310, Balance:  1.03

Timing Information --------------------------------------------------
  I/O:                          0.820
  Partitioning:                 1.750   (KMETIS time)
  Total:                        2.570
**********************************************************************
```

Figure 3: Output of `pmetis` and `kmetis` for graph *brack2.graph* and a 100-way partition.

4.2 Mesh Partitioning Programs

METIS provides two programs `partnmesh` and `partdmesh` for partitioning meshes (*e.g.*, those arising in finite element or finite volume methods) into *k* equal size parts. These programs take as input the element node array of the mesh and compute a partitioning for both its elements and its nodes. METIS currently supports four different types of mesh elements which are triangles, tetrahedra, hexahedra (bricks), and quadrilaterals.

These programs first convert the mesh into a graph, and then use `kmetis` to partition this graph. The difference between these two programs is that `partnmesh` converts the mesh into a nodal graph (*i.e.*, each node of the mesh becomes a vertex of the graph), whereas `partdmesh` converts the mesh into a dual graph (*i.e.*, each element becomes a vertex of the graph). In the case of `partnmesh`, the partitioning of the nodal graph is used to derive a partitioning of the elements. In the case of `partdmesh`, the partitioning of the dual graph is used to derive a partitioning of the nodes. Both of these programs produce partitioning of comparable quality, with `partnmesh` being considerably faster than `partdmesh`. However, in some cases, `partnmesh` may produce partitions that have higher load imbalance than `partdmesh`.

Both `partnmesh` and `partdmesh` are invoked by providing two arguments at the command line as follows:

partnmesh *MeshFile* *Nparts*
partdmesh *MeshFile* *Nparts*

The first argument *MeshFile*, is the name of the file that stores the mesh (whose format is described in Section 4.5.2), while the second argument *Nparts*, is the number of partitions that is desired. Both `partnmesh` and `partdmesh` can partition a mesh into an arbitrary number of partitions. Upon successful execution, both programs display statistics regarding the quality of the computed partitioning and the amount of time taken to perform the partitioning. The

```
prompt% pmetis m14.graph3 16
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: m14.graph3, #Vertices: 214765, #Edges: 1679018, #Parts: 16
  Balancing Constraints: 3

Recursive Partitioning... -------------------------------------------
  16-way Edge-Cut:   74454, Balance:  1.01  1.03  1.02

Timing Information --------------------------------------------------
  I/O:                            4.310
  Partitioning:                  28.410   (PMETIS time)
  Total:                         32.830
**********************************************************************

prompt% kmetis m14.graph3 16
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: m14.graph3, #Vertices: 214765, #Edges: 1679018, #Parts: 16
  Balancing Constraints: 3

K-way Partitioning... -----------------------------------------------
  16-way Edge-Cut:   71410, Balance:  1.04  1.04  1.04

Timing Information --------------------------------------------------
  I/O:                            4.020
  Partitioning:                   7.430   (KMETIS time)
  Total:                         11.550
**********************************************************************
```

Figure 4: Output of `pmetis` and `kmetis` for a multi-constraint graph with three constraints and a 16-way partition.

actual partitioning is stored in two files named: ***MeshFile.npart.Nparts*** which stores the partitioning of the nodes, and ***MeshFile.epart.Nparts*** which stores the partitioning of the elements. The format of the partitioning files is described in Section 4.6.1.

Figure 5 shows the output of `partnmesh` and `partdmesh` for partitioning a mesh with tetrahedron elements into 100 parts. From this figure we see that both programs initially print information about the mesh, such as its name, the number of elements (*#Elements*), the number of nodes (*#Nodes*), and the type of elements (*e.g.*, *TET*). Next, they print some information regarding the quality of the partitioning. Specifically, they report the number of edges being cut (*Edge-Cut*) by the partitioning[2], as well as the balance of the partitioning. For both `partnmesh` and `partdmesh`, the balance is computed with respect to the number of elements. The balance with respect to the number of nodes is not shown, but it is in general similar to the element balance.

Finally, both `partnmesh` and `partdmesh` show the time that was taken by the various phases of the algorithm. All times are in seconds. In this particular example, it took `partnmesh` 23.370 seconds to partition the mesh into 100 parts. Note that this time includes the time required both to construct the nodal graph and to partition it. Similarly, it took `partdmesh` 74.560 seconds to partition the same mesh. Again, this time includes the time required both to construct the dual graph and to partition it. As you can see from this example, `partnmesh` is considerably faster than `partdmesh`. This is because of two reasons: (i) the time required to construct the nodal graph is smaller than the time required to construct the dual graph; (ii) the nodal graph is smaller than the dual graph.

[2]The edgecut that is reported by `partnmesh` is that of the nodal graph, whereas the edgecut reported by `partdmesh` is that of the dual graph. These two edgecuts cannot be compared with each other, as they correspond to partitionings of two totally different graphs.

Note If you need to compute multiple partitionings of the same mesh, it may be preferable to first use one of the mesh conversion programs described in Section 4.4 to first convert the mesh into a graph, and then use `kmetis` to partition it. By doing this, you pay the cost of converting the mesh into a graph only once.

```
prompt% partnmesh 144.mesh 100
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Mesh Information ----------------------------------------------------
  Name: 144.mesh, #Elements: 905410, #Nodes: 144649, Etype: TET

Partitioning Nodal Graph... -----------------------------------------
  100-way Edge-Cut:  105207, Balance:  1.03

Timing Information --------------------------------------------------
  I/O:                          13.210
  Partitioning:                  7.950
**********************************************************************

prompt% partdmesh 144.mesh 100
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Mesh Information ----------------------------------------------------
  Name: 144.mesh, #Elements: 905410, #Nodes: 144649, Etype: TET

Partitioning Dual Graph... ------------------------------------------
  100-way Edge-Cut:   52474, Balance:  1.03

Timing Information --------------------------------------------------
  I/O:                          11.540
  Partitioning:                 28.220
**********************************************************************
```

Figure 5: Output of `partnmesh` and `partdmesh` for mesh *144.mesh* and a 100-way partition.

4.3 Sparse Matrix Reordering Programs

METIS provides two programs `oemetis` and `onmetis` for computing fill-reducing orderings of sparse matrices. Both of these programs use multilevel nested dissection to compute a fill-reducing ordering [8]. The nested dissection paradigm is based on computing a vertex-separator for the the graph corresponding to the matrix. The nodes in the separator are moved to the end of the matrix, and a similar process is applied recursively for each one of the other two parts.

Even though both programs are based on multilevel nested dissection, they differ on how they compute the vertex separators. The `oemetis` program finds a vertex separator by first computing an edge separator using a multilevel algorithm, whereas the `onmetis` program uses the multilevel paradigm to directly find a vertex separator. The orderings produced by `onmetis` generally incur less fill than those produced by `oemetis`. In particular, for matrices arising in linear programming problems the orderings computed by `onmetis` are significantly better than those produced by `oemetis`. Furthermore, `onmetis` utilizes compression techniques to reduce the size of the graph prior to computing the ordering. Sparse matrices arising in many application domains are such that certain rows of the matrix have the same sparsity pattern. Such matrices can be represented by a much smaller graph in which all rows with identical sparsity pattern are represented by just a single vertex whose weight is equal to the number of rows. Such compression techniques can significantly reduce the size of the graph, whenever applicable, and substantially reduce the amount of time required by `onmetis`. However, when there is no reduction in graph size, `oemetis` is about 20% to 30% faster than `onmetis`. Furthermore, for large matrices arising in three-dimensional problems, the quality

```
prompt% oemetis bcsstk31.graph
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: bcsstk31.graph, #Vertices: 35588, #Edges: 572914

Edge-Based Ordering... ----------------------------------------------
  Nonzeros: 4693428,    Operation Count: 1.4356e+09

Timing Information --------------------------------------------------
  I/O:                             1.160
  Ordering:                        7.380   (OEMETIS time)
  Symbolic Factorization:          0.440
  Total:                           8.980
**********************************************************************

prompt% onmetis bcsstk31.graph
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Graph Information ---------------------------------------------------
  Name: bcsstk31.graph, #Vertices: 35588, #Edges: 572914

Node-Based Ordering... ----------------------------------------------
  Nonzeros: 4330669,    Operation Count: 1.1564e+09

Timing Information --------------------------------------------------
  I/O:                             1.080
  Ordering:                        4.540   (ONMETIS time)
  Symbolic Factorization:          0.440
  Total:                           6.060
**********************************************************************
```

Figure 6: Output of `oemetis` and `onmetis` for graph *bcsstk31.graph*.

of orderings produced by the two algorithms is quite similar.

Both `oemetis` and `onmetis` are invoked by providing one argument at the command line as follows:

oemetis *GraphFile*
onmetis *GraphFile*

The only argument of these programs *GraphFile*, is the name of the file that stores the sparse matrix in the graph format described in Section 4.5.1. Upon successful execution, both programs display statistics regarding the quality of the computed orderings and the amount of time taken to perform the ordering. The actual ordering is stored in a file named ***GraphFile.iperm***, whose format is described in Section 4.6.2.

Figure 6 shows the output of `oemetis` and `onmetis` for computing a fill-reducing ordering of a sample matrix. From this figure we see that both programs initially print information about the graph, such as its name, the number of vertices (*#Vertices*), and the number of edges (*#Edges*). Next, they print some information regarding the quality of the ordering. Specifically, they report the number of non-zeros that are required in the lower triangular matrix, and the number of operations (*OPC*) required to factor the matrix using Cholesky factorization. Note that number of nonzeros includes both the original non-zeros and the new non-zeros due to the fill. Finally, both `oemetis` and `onmetis` show the time that was taken by the various phases of the algorithm. All times are in seconds. For this particular example, `oemetis` takes a total of 23.290 seconds, of which 17.760 seconds was taken by the ordering algorithm itself. For the same example `onmetis` takes a total of 17.340 seconds, of which 11.810 seconds was taken by the partitioning algorithm itself. Note that in this case `onmetis` is faster than `oemetis`, because `onmetis` was able to compress the matrix. Also note that the quality of the fill-reducing ordering produced by `onmetis` is significantly better than that produced by `oemetis`. In fact, the ordering produced by `onmetis` results in 8% fewer non-zeros and 20% fewer operations.

4.4 Auxiliary Programs

4.4.1 Mesh To Graph Conversion

```
prompt% mesh2nodal 144.mesh
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Mesh Information ----------------------------------------------------
  Name: 144.mesh, #Elements: 905410, #Nodes: 144649, Etype: TET

Forming Nodal Graph... ----------------------------------------------
  Nodal Information: #Vertices: 144649, #Edges: 1074393

Timing Information --------------------------------------------------
  I/O:                          15.290
  Nodal Creation:                3.030
**********************************************************************

prompt% mesh2dual 144.mesh
**********************************************************************
  METIS 4.0   Copyright 1998, Regents of the University of Minnesota

Mesh Information ----------------------------------------------------
  Name: 144.mesh, #Elements: 905410, #Nodes: 144649, Etype: TET

Forming Dual Graph... -----------------------------------------------
  Dual Information: #Vertices: 905410, #Edges: 1786484

Timing Information --------------------------------------------------
  I/O:                          19.200
  Dual Creation:                10.880
**********************************************************************
```

Figure 7: Output of `mesh2nodal` and `mesh2dual` for mesh *144.mesh*.

METIS provides two programs `mesh2nodal` and `mesh2dual` for converting a mesh into the graph format used by METIS. In particular, `mesh2nodal` converts the element node array of a mesh into a nodal graph; *i.e.*, each node of the mesh corresponds to a vertex in the graph and two vertices are connected by an edge if the corresponding nodes are connected by lines in the mesh. Similarly, `mesh2dual` converts the element node array of a mesh into a dual graph; *i.e.*, each element of the mesh corresponds to a vertex in the graph and two vertices are connected if the corresponding elements in the mesh share a face. These mesh-to-graph conversion programs support meshes with triangular, tetrahedra, and hexahedra (bricks) elements.

Both `mesh2nodal` and `mesh2dual` are invoked by providing one argument at the command line as follows:

mesh2nodal *MeshFile*
mesh2dual *MeshFile*

The only argument of these programs *MeshFile*, is the name of the file that stores the mesh (whose format is described in Section 4.5.2). Upon successful execution, both programs display information about the generated graphs, and the amount of time taken to perform the conversion. The actual graph is stored in a file named: ***MeshFile.ngraph*** in the case of `mesh2nodal` and ***MeshFile.dgraph*** in the case of `mesh2dual`. The format of these graph files are compatible with METIS and is described in Section 4.5.1.

Figure 7 shows the output of `mesh2nodal` and `mesh2dual` for generating the nodal and dual graphs of a sample mesh. Note that the sizes of the generated graphs are different, as the dual graph is larger than the nodal graph. Also note that generating the nodal graph is considerably faster than generating the dual graph.

4.4.2 Graph Checker

METIS provide a program called `graphchk` to check whether or not the format of a graph is appropriate for use with METIS. This program should be used whenever there is any doubt about the format of any graph file. It is invoked by providing one argument at the command line as follows:

graphchk *GraphFile*

where *GraphFile* is the name of the file that stores the graph.

4.5 Input File Formats

The various programs in METIS require as input either a file storing a graph or a file storing a mesh. The format of these files are described in the following sections.

4.5.1 Graph File

The primary input of the partitioning and fill-reducing ordering programs in METIS is the graph to be partitioned or ordered. This graph is stored in a file and is supplied to the various programs as one of the command line parameters.

A graph $G = (V, E)$ with n vertices and m edges is stored in a plain text file that contains $n + 1$ lines (excluding comment lines). The first line contains information about the size and the type of the graph, while the remaining n lines contain information for each vertex of G. Any line that starts with '%' is a comment line and is skipped.

The first line contains either two (n, m), three (n, m, *fmt*), or four (n, m, *fmt*, *ncon*) integers. The first two integers (n, m) are the number of vertices and the number of edges, respectively. Note that in determining the number of edges m, an edge between any pair of vertices v and u is counted **only once** and not twice (*i.e.*, we do not count the edge (v, u) separately from (u, v)). For example, the graph in Figure 8 contains 11 vertices. The third integer (*fmt*) is used to specify whether or not the graph has weights associated with its vertices, its edges, or both. Table 2 describes the possible values of *fmt* and their meaning. Note that if the graph is unweighted (*i.e.*, all vertices and edges have the same weight), then the *fmt* parameter can be omitted. Finally, the fourth integer (*ncon*) is used to specify the number of weights associated with each vertex of the graph. The value of this parameter determines whether or not METIS will use the multi-constraint partitioning algorithms described in Section 3. If the vertices of the graph have no weights or only a single weight, then the *ncon* parameter can be omitted. However, if *ncon* is greater than 0, then the file should contain the required vertex weights and the *fmt* parameter should be set appropriately (*i.e.*, it should be set to either 10 or 11).

fmt	Meaning
0	The graph has no weights associated with either the edges or the vertices
1	The graph has weights associated with the edges
10	The graph has weights associated with the vertices
11	The graph has weights associated with both the edges & vertices

Table 2: The various possible values for the *fmt* parameter and their meaning.

The remaining n lines store information about the actual structure of the graph. In particular, the ith line (excluding comment lines) contains information that is relevant to the ith vertex. Depending on the value of the *fmt* and *ncon* parameters, the information stored at each line is somewhat different. In the most general form (when *fmt = 11* and *ncon > 1*) each line will have the following structure:

$$w_1, w_2, \ldots w_{ncon},\ v_1, e_1, v_2, e_2, \ldots, v_k, e_k$$

where $w_1, w_2, \ldots, w_{ncon}$ are the *ncon* vertex weights associated with this vertex, $v_1, v_2, \ldots, v_k$ are the vertices adjacent to this vertex, and $e_1, e_2, \ldots, e_k$ are the weights of these edges. In the remaining of this section we illustrate this format by a sequence of examples. Note that the vertices are numbered starting from 1 (not from 0 as is often done in C). Furthermore, the vertex-weights must be integers greater or equal to 0, whereas the edge-weights must be strictly greater than 0.

The simplest format for a graph G is when the weight of all vertices and the weight of all the edges is the same. This format is illustrated in Figure 8(a). Note, the optional *fmt* parameter is skipped in this case.

However, there are cases in which the edges in G have different weights. This is accommodated as shown in Figure 8(b). Now, the adjacency list of each vertex contains the weight of the edges in addition to the vertices that is connected with. If v has k vertices adjacent to it, then the line for v in the graph file contains $2 * k$ numbers, each pair of numbers stores the vertex that v is connected to, and the weight of the edge. Note that the *fmt* parameter is equal

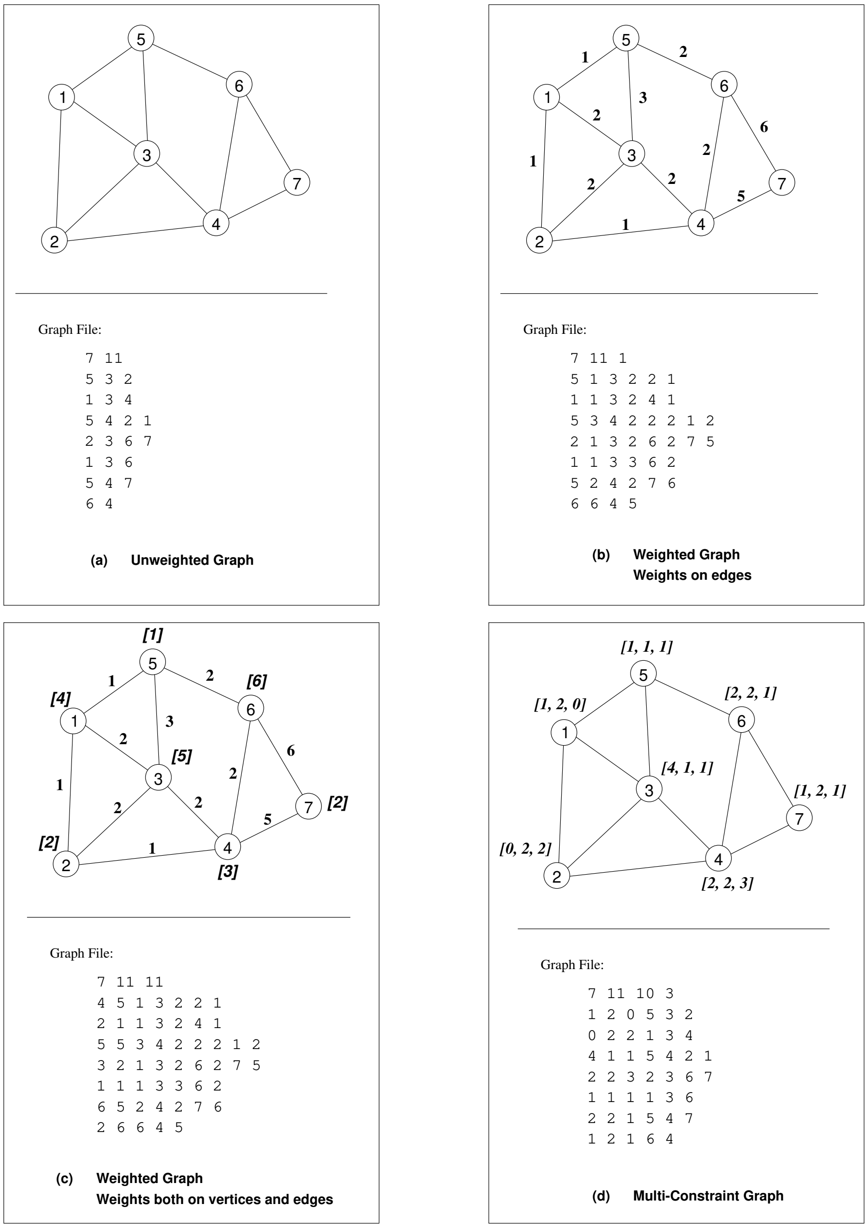


Figure 8: Storage format for various type of graphs.

to 1, indicating the fact that G has weights on the edges.

In addition to having weights on the edges, weights on the vertices are also allowed, as illustrated in Figure 8(c). In this case, the value of *fmt* is equal to 11, and each line of the graph file first stores the weight of the vertex, and then the weighted adjacency list.

Finally, Figure 8(d) illustrates the format of the input file when the vertices of the graph contain multiple weights (3 in this example). In this case, the value of *fmt* is equal to 10 (we do not have weights associated with the edges), and the value of *ncon* is equal to 3 (since we have three sets of vertex-weights). Each line of the graph file stores the three weights of the vertices followed by the adjacency list.

4.5.2 Mesh File

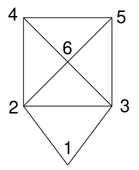
The primary input of the mesh partitioning programs in METIS is the mesh to be partitioned. This mesh is stored in a file in the form of the element node array. A mesh with n elements is stored in a plain text file that contains $n + 1$

lines. The first line contains information about the size and the type of the mesh, while the remaining n lines contain the nodes that make up each element.

The first line contains two integers. The first integer is the number of elements n in the mesh. The second integer *etype* is used to denote the type of elements that the mesh is made off. *Etype* can either take the values of 1, 2, 3, or 4, indicating that the mesh consists of either triangles, tetrahedra, hexahedra (bricks), or quadrilaterals, respectively.

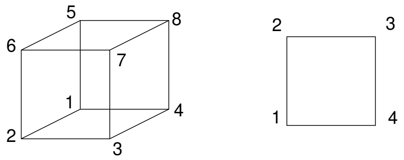
After the first line, the remaining n lines store the element node array. In particular for element i, line $i + 1$ stores the nodes that this element is made off. Depending on *etype*, each line can either have three integers (in the case of triangles), four integers (in the case of tetrahedra and quadrilaterals), or eight integers (in the case of hexahedra). In the case of triangles and tetrahedra, the ordering of the nodes for each element does not matter. However, in the case of hexahedra and quadrilaterals, the nodes for each element should be ordered according to the numbering illustrated in Figure 9(b). Note that the node numbering starts from 1.

Figure 9 illustrates this format for a small mesh with triangular elements. Note that the *etype* field of the mesh file is set to 1 indicating that the mesh consists of triangular elements.



Mesh File:
```
5 1
1 2 3
2 4 6
2 6 3
4 5 6
5 6 3
```

(a) Sample Mesh File



(b) Ordering of nodes

Figure 9: (a) The file that stores the mesh. (b) The ordering of the nodes in the case of hexahedra and quadrilaterals.

4.6 Output File Formats

The output of METIS is either a partition or an ordering file, depending on whether METIS is used for graph/mesh partitioning or for sparse matrix ordering. The format of these files are described in the following sections.

4.6.1 Partition File

The partition file of a graph with n vertices consists of n lines with a single number per line. The ith line of the file contains the partition number that the ith vertex belongs to. Partition numbers start from 0 up to the number of partitions minus one.

4.6.2 Ordering File

The ordering file of a graph with n vertices consists of n lines with a single number per line. The ith line of the ordering file contains the new order of the ith vertex of the graph. The numbering in the ordering file starts from 0.

Note that the ordering file stores what is referred to as the the inverse permutation vector *iperm* of the ordering. Let A be a matrix and let A' be the reordered matrix. The inverse permutation vector maps the ith row (column) of A into the *iperm*$[i]$ row (column) of A'.

5 METIS's Library Interface

The various programs provided in METIS can also be directly accessed from a C or Fortran program by using the stand-alone library METISlib. Furthermore, METISlib extends the functionality provided by METIS's stand-alone programs in two different ways. First, it allows the user to alter the behavior of the various algorithms in METIS, and second METISlib provides additional routines that can be used to partition graphs into unequal-size partitions and compute partitionings that directly minimize the total communication volume.

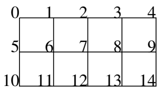
In the rest of this section we describe the interface to the routines in METISlib by first describing the various data structures used to pass information into and get information out of the routines, followed by a detailed description of the calling sequence of the various routines.

5.1 Graph Data Structure

All of the graph partitioning and sparse matrix ordering routines in METISlib take as input the adjacency structure of the graph and the weights of the vertices and edges (if any).

The adjacency structure of the graph is stored using the compressed storage format (CSR). The CSR format is a widely used scheme for storing sparse graphs. In this format the adjacency structure of a graph with n vertices and m edges is represented using two arrays `xadj` and `adjncy`. The `xadj` array is of size $n+1$ whereas the `adjncy` array is of size $2m$ (this is because for each edge between vertices v and u we actually store both (v, u) and (u, v)).

The adjacency structure of the graph is stored as follows. Assuming that vertex numbering starts from 0 (C style), then the adjacency list of vertex i is stored in array `adjncy` starting at index `xadj`[i] and ending at (but not including) index `xadj`[$i+1$] (*i.e.*, `adjncy`[`xadj`[i]] through and including `adjncy`[`xadj`[$i+1$]-1]). That is, for each vertex i, its adjacency list is stored in consecutive locations in the array `adjncy`, and the array `xadj` is used to point to where it begins and where it ends. Figure 10(b) illustrates the CSR format for the 15-vertex graph shown in Figure 10(a).



(a) A sample graph

xadj	0 2 5 8 11 13 16 20 24 28 31 33 36 39 42 44
adjncy	1 5 0 2 6 1 3 7 2 4 8 3 9 0 6 10 1 5 7 11 2 6 8 12 3 7 9 13 4 8 14 5 11 6 10 12 7 11 13 8 12 14 9 13

(b CSR format

Figure 10: An example of the CSR format for storing sparse graphs.

The weights of the vertices (if any) are stored in an additional array called `vwgt`. If *ncon* is the number of weights associated with each vertex, the array `vwgt` contains $n * ncon$ elements (recall that n is the number of vertices). The weights of the ith vertex are stored in *ncon* consecutive entries starting at location `vwgt`[i * `ncon`]. Note that if each vertex has only a single weight, then `vwgt` will contain n elements, and `vwgt`[i] will store the weight of the ith vertex. The vertex-weights must be integers greater or equal to zero. If all the vertices of the graph have the same weight (*i.e.*, the graph is unweighted), then the `vwgt` can be set to NULL.

The weights of the edges (if any) are stored in an additional array called `adjwgt`. This array contains $2m$ elements, and the weight of edge `adjncy`[j] is stored at location `adjwgt`[j]. The edge-weights must be integers greater than zero. If all the edges of the graph have the same weight (*i.e.*, the graph is unweighted), then the `adjwgt` can be set to NULL.

All of these four arrays (*xadj*, *adjncy*, *vwgt*, and *adjwgt*) are defined in METISlib to be of of type `idxtype`. By default `idxtype` is set to be equivalent to type `int` (*i.e.*, the integer datatype of C). However, `idxtype` can be made to be equivalent to a `short int` for certain architectures that use 64-bit integers by default. The conversion of `idxtype` from `int` to `short` can be done by modifying the file `Lib/struct.h` (instructions are included there). The same `idxtype` is used for the arrays that are used to store the computed partition and permutation vector.

5.2 Mesh Data Structure

All of the mesh partitioning and mesh conversion routines in METISlib take as input the element node array of a mesh. This element node array is stored using an array called `elmnts`. For a mesh with n elements and k nodes per element, the size of the `elmnts` array is $n * k$. Note that since the supported elements in METIS are only triangles, tetrahedra, hexahedra, and quadrilaterals, the possible values for k are 3, 4, 8, and 4, respectively.

The element node array of the mesh is stored in `elmnts` as follows. Assuming that the element numbering starts from 0 (C style), then the k nodes that make up element i are stored in array `elmnts` starting at index $i * k$ and ending (but not including) index $(i + 1) * k$. As it was the case with the format of the mesh file described in Section 4.5.2, the ordering of the nodes is not important for triangle and tetrahedra elements. However, in the case of hexahedra, the nodes for each element must be ordered according to the numbering illustrated in Figure 9(b).

The array that describes the element node array of the mesh is defined in METISlib to be of type `idxtype`, which by default is equivalent to `int` (*i.e.*, integers).

5.3 Partitioning Objectives

The partitioning algorithms in METISlib can be used to compute a balanced k-way partitioning that minimizes either the number of edges that straddle partitions (*edgecut*) or the total communication volume (*totalv*). In the rest of this section we briefly describe these two objectives and provide some suggestions on when they should be used.

Minimizing the Edge-Cut Consider a graph $G = (V, E)$, and let P be a vector of size $|V|$ such that $P[i]$ stores the number of the partition that vertex i belongs to. The *edgecut* of this partitioning is defined as the number of edges that straddle partitions. That is, the number of edges (v, u) for which $P[v] \neq P[u]$. If the graph has weights associated with the edges, then the edgecut is defined as the sum of the weight of these straddling edges.

Minimizing the Total Communication Volume Consider a graph $G = (V, E)$, and let P be a vector of size $|V|$ such that $P[i]$ stores the number of the partition that vertex i belongs to. Let $V_b \subset V$ be the subset of interface (or boarder) vertices. That is, each vertex $v \in V_b$ is connected to at least one vertex that belongs to a different partition. For each vertex $v \in V_b$ let $Nadj[v]$ be the number of domains other than $P[v]$ that the vertices adjacent to v belong to. The *totalv* of this partitioning is defined as:

$$totalv = \sum_{v \in V_b} Nadj[v]. \tag{1}$$

Equation 1 corresponds to the total communication volume incurred by the partitioning because each interface vertex v needs to be sent to all of its $Nadj[v]$ partitions.

The above model can be extended to instances in which the amount of data that needs to be sent for each node is different. In particular, if w_v is the amount of data that needs to be sent for vertex v, then Equation 1 can be re-written as:

$$totalv = \sum_{v \in V_b} w_v Nadj[v]. \tag{2}$$

METISlib supports this weighted totalv model by using an array called `vsize` such that the amount of data that needs to be sent due to the ith vertex is stored in `vsize[i]`. Note that the amount of data that needs to be sent is different from the *weight* of the vertex. The former corresponds to communication cost whereas the later corresponds to the computational cost.

Note that for partitioning algorithms to correctly minimize the totalv, the graph should reflect the true information exchange requirements of the underlying computations. For instance, the dual graph of a finite element mesh does not correctly model the underlying communication, whereas the nodal graph does.

Which one is Better? When partitioning is used to distribute a graph or a mesh among the processors of a parallel computer, the edgecut is only an approximation of the true communication cost resulting from the partitioning. On the other hand, by minimizing the totalv we can directly minimize the overall communication cost. Despite of that, for many graphs the solutions obtained by minimizing the edgecut or minimizing the totalv, are comparable. This is especially true for graphs corresponding to well-shaped finite element meshes. This is because for these graphs, the degrees of the various vertices are similar and the objectives of minimizing the edgecut or the totalv behave the same. On the other hand, if the vertex degrees vary significantly (*e.g.*, graphs corresponding to linear programming matrices), then by minimizing the totalv we can obtain a significant reduction in the total communication volume.

In terms of the amount of time required by these two partitioning objectives, minimizing the edgecut is faster than minimizing the totalv. For this reason, the totalv objective should be used only for problems in which it actually reduces the overall communication volume.

5.4 Graph Partitioning Routines

METIS_PartGraphRecursive (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k equal-size parts using multilevel recursive bisection. It provides the functionality of the `pmetis` program. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

- 0 No weights (vwgts and adjwgt are NULL)
- 1 Weights on the edges only (vwgts = NULL)
- 2 Weights on the vertices only (adjwgt = NULL)
- 3 Weights both on vertices and edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

- 0 C-style numbering is assumed that starts from 0
- 1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines matching type. Possible values are:

- 1 Random Matching (RM)
- 2 Heavy-Edge Matching (HEM)
- 3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

- 1 Region Growing (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

- 1 Early-Exit Boundary FM refinement (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a small number of partitions (less than 8). If a large number of partitions is desired, the METIS_PartGraphKway should be used instead, as it is significantly faster.

METIS_PartGraphKway (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k equal-size parts using the multilevel k-way partitioning algorithm. It provides the functionality of the `kmetis` program. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

- 0 No weights (vwgts and adjwgt are NULL)
- 1 Weights on the edges only (vwgts = NULL)
- 2 Weights on the vertices only (adjwgt = NULL)
- 3 Weights both on vertices and edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

- 0 C-style numbering is assumed that starts from 0
- 1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

- 1 Random Matching (RM)
- 2 Heavy-Edge Matching (HEM)
- 3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

- 1 Multilevel recursive bisection (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

- 1 Random boundary refinement
- 2 Greedy boundary refinement
- 3 Random boundary refinement that also minimizes the connectivity among the subdomains (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size n that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a large number of partitions (greater than 8). If a small number of partitions is desired, the METIS_PartGraphRecursive should be used instead, as it produces somewhat better partitions.

METIS_PartGraphVKway (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *vsize, int *wgtflag, int *numflag, int *nparts, int *options, int *volume, idxtype *part)

Description

It is used to partition a graph into k equal-size parts using the multilevel k-way partitioning algorithm. The objective of the partitioning is to minimize the total communication volume (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Sections 5.1 and 5.3.

vwgt, vsize
Information about the weights of the vertices related to the computation and communication as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

0 No weights (vwgts and vsize are NULL)
1 Communication weights only (vwgts = NULL)
2 Computation weights only (vsize = NULL)
3 Both communication and computation weights.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0
1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:
1 Random Matching (RM)
2 Heavy-Edge Matching (HEM)
3 Sorted Heavy-Edge Matching (SHEM) (Default)
Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:
1 Multilevel recursive bisection (Default)

options[3] Determines the algorithm used for refinement. Possible values are:
1 Random boundary refinement (Default)
3 Random boundary refinement that also minimizes the connectivity among the subdomains

options[4] Used for debugging purposes. Always set it to 0 (Default).

volume Upon successful completion, this variable stores the total communication volume requires by the partition.

part This is a vector of size n that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

METIS_mCPartGraphRecursive (int *n, int *ncon, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k parts such that multiple balancing constraints are satisfied. It uses the multi-constraint multilevel recursive bisection algorithm. It provides the functionality of the `pmetis` program when it is used to compute a multi-constraint partitioning. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

ncon The number of constraints. This should be greater than one and smaller than 15.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1. Note that the weight vector must be supplied and it should be of size `n*ncon`.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

0 No weights (adjwgt is NULL)

1 Weights on the edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

5 Sorted Heavy-Edge Matching followed by 1-norm Balanced-edge (SHEBM1N)

6 Sorted Heavy-Edge Matching followed by ∞-norm Balanced-edge (SHEBMIN) (Default)

7 1-norm Balanced-edge followed by Heavy-Edge Matching (SBHEM1N)

8 ∞-norm Balanced-edge followed by Heavy-Edge Matching (SBHEMIN)

Experiments has shown that for simple balancing problems, the schemes that give priority to heavy edges (*e.g.*, SHEM, SHEBM1N, SHEBMIN) perform better, and for hard balancing problems, the schemes that give priority to balanced edges (*e.g.*, SBHEM1N, SBHEMIN) perform better.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Multi-constraint Greedy Graph Growing

2 Random (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

1 Early-Exit Boundary FM refinement (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a small number of partitions. If a large number of partitions is desired, the METIS_mCPartGraphKway should be used instead, as it produces somewhat better partitions (both in terms of quality and balance).

METIS_mCPartGraphKway (int *n, int *ncon, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, float *ubvec, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k parts such that multiple balancing constraints are satisfied. It uses the multi-constraint multilevel k-way partitioning algorithm. It provides the functionality of the `kmetis` program when it is used to compute a multi-constraint partitioning. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

ncon The number of constraints. This should be greater than one and smaller than 15.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1. Note that the weight vector must be supplied and it should be of size `n*ncon`.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

0 No weights (adjwgt is NULL)

1 Weights on the edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

ubvec This is a vector of size `ncon` that specifies the load imbalance tolerances for each one of the `ncon` constraints. Each tolerance should be greater than 1.0 (preferably greater than 1.03).

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

5 Sorted Heavy-Edge Matching followed by 1-norm Balanced-edge (SHEBM1N)

6 Sorted Heavy-Edge Matching followed by ∞-norm Balanced-edge (SHEBMIN) (Default)

7 1-norm Balanced-edge followed by Heavy-Edge Matching (SBHEM1N)

8 ∞-norm Balanced-edge followed by Heavy-Edge Matching (SBHEMIN)

Experiments has shown that for simple balancing problems, the schemes that give priority to heavy edges (*e.g.*, SHEM, SHEBM1N, SHEBMIN) perform better, and for hard balancing problems, the schemes that give priority to balanced edges (*e.g.*, SBHEM1N, SBHEMIN) perform better.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Multilevel recursive bisection

2 Relaxed Multilevel recursive bisection (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

1 Random boundary refinement (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a large number of partitions (greater than 8). If a small number of partitions is desired, the METIS_mCPartGraphRecursive should be used instead, as it produces somewhat better partitions.

METIS_WPartGraphRecursive (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, float *tpwgts, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k parts using multilevel recursive bisection. The underlying algorithm is similar to the one used by METIS_PartGraphRecursive, but it can be used to compute a partitioning with prescribed partition weights. For example, it can be used to compute a 3-way partition such that partition 1 has 50% of the weight, partition 2 has 20% of the weight, and partition 3 has 30% of the weight. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

- 0 No weights (vwgts and adjwgt are NULL)
- 1 Weights on the edges only (vwgts = NULL)
- 2 Weights on the vertices only (adjwgt = NULL)
- 3 Weights both on vertices and edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

- 0 C-style numbering is assumed that starts from 0
- 1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

tpwgts This is an array containing *nparts* floating point numbers. For partition i, *tpwgts[i]* stores the fraction of the total weight that should be assigned to it. For example, for a 4-way partition the vector *tpwgts[]* = {*0.2 0.2 0.4 0.2*} will result in partitions 0, 1, and 3 having 20% of the weight and partition 2 having 40% of the weight. Note that the numbers in *tpwgts* should add up to 1.0.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

- 1 Random Matching (RM)
- 2 Heavy-Edge Matching (HEM)
- 3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

- 1 Region Growing (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

- 1 Early-Exit Boundary FM refinement (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a small number of partitions (less than 8). If a large number of partitions is desired, the METIS_WPartGraphKway should be used instead, as it is significantly faster.

METIS_WPartGraphKway (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *adjwgt, int *wgtflag, int *numflag, int *nparts, float *tpwgts, int *options, int *edgecut, idxtype *part)

Description

It is used to partition a graph into k parts using multilevel recursive bisection. The underlying algorithm is similar to the one used by METIS_PartGraphKway, but it can be used to compute a partitioning with prescribed partition weights. For example, it can be used to compute a 3-way partition such that partition 1 has 50% of the weight, partition 2 has 20% of the weight, and partition 3 has 30% of the weight. The objective of the partitioning is to minimize the edgecut (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt, adjwgt
Information about the weights of the vertices and edges as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

- 0 No weights (vwgts and adjwgt are NULL)
- 1 Weights on the edges only (vwgts = NULL)
- 2 Weights on the vertices only (adjwgt = NULL)
- 3 Weights both on vertices and edges.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

- 0 C-style numbering is assumed that starts from 0
- 1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

tpwgts This is an array containing *nparts* floating point numbers. For partition i, *tpwgts[i]* stores the fraction of the total weight that should be assigned to it. For example, for a 4-way partition the vector *tpwgts[]* = {*0.2 0.2 0.4 0.2*} will result in partitions 0, 1, and 3 having 20% of the weight and partition 2 having 40% of the weight. Note that the numbers in *tpwgts* should add up to 1.0.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

- 1 Random Matching (RM)
- 2 Heavy-Edge Matching (HEM)
- 3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

- 1 Multilevel recursive bisection (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

- 1 Random boundary refinement
- 2 Greedy boundary refinement
- 3 Random boundary refinement that also minimizes the connectivity among the subdomains (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function should be used to partition a graph into a large number of partitions (greater than 8). If a small number of partitions is desired, the METIS_WPartGraphRecursive should be used instead, as it produces somewhat better partitions.

METIS_WPartGraphVKway (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, idxtype *vsize, int *wgtflag, int *numflag, int *nparts, float *tpwgts, int *options, int *volume, idxtype *part)

Description

It is used to partition a graph into k parts using multilevel recursive bisection. The underlying algorithm is similar to the one used by METIS_PartGraphKway, but it can be used to compute a partitioning with prescribed partition weights. For example, it can be used to compute a 3-way partition such that partition 1 has 50% of the weight, partition 2 has 20% of the weight, and partition 3 has 30% of the weight. The objective of the partitioning is to minimize the total communication volume (as described in Section 5.3).

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Sections 5.1 and 5.3.

vwgt, vsize
Information about the weights of the vertices related to the computation and communication as described in Section 5.1.

wgtflag Used to indicate if the graph is weighted. *wgtflag* can take the following values:

0 No weights (vwgts and vsize are NULL)

1 Communication weights only (vwgts = NULL)

2 Computation weights only (vsize = NULL)

3 Both communication and computation weights.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the graph.

tpwgts This is an array containing *nparts* floating point numbers. For partition i, *tpwgts[i]* stores the fraction of the total weight that should be assigned to it. For example, for a 4-way partition the vector *tpwgts[]* = {*0.2 0.2 0.4 0.2*} will result in partitions 0, 1, and 3 having 20% of the weight and partition 2 having 40% of the weight. Note that the numbers in *tpwgts* should add up to 1.0.

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Multilevel recursive bisection (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

1 Random boundary refinement (Default)

3 Random boundary refinement that also minimizes the connectivity among the subdomains

options[4] Used for debugging purposes. Always set it to 0 (Default).

volume Upon successful completion, this variable stores the total communication volume required by the partition.

part This is a vector of size *n* that upon successful completion stores the partition vector of the graph. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

5.5 Mesh Partitioning Routines

METIS_PartMeshNodal (int *ne, int *nn, idxtype *elmnts, int *etype, int *numflag, int *nparts, int *edgecut, idxtype *epart, idxtype *npart)

Description

This function is used to partition a mesh into k equal-size parts. It provides the functionality of the `partnmesh` program.

Parameters

ne The number of elements in the mesh.

nn The number of nodes in the mesh.

elmnts The element node array storing the mesh as described in Section 5.2.

etype Indicates the type of the elements in the mesh. *etype* can take the following values:

1 The elements are triangles.
2 The elements are tetrahedra.
3 The elements are hexahedra (bricks).
4 The elements are quadrilaterals.

numflag Used to indicate which numbering scheme is used for the element node array. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0
1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the mesh.

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition in the nodal graph.

epart This is a vector of size *ne* that upon successful completion stores the partition vector for the elements of the mesh. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

npart This is a vector of size *nn* that upon successful completion stores the partition vector for the nodes of the mesh. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function converts the mesh into a nodal graph and then uses METIS_PartGraphKway to compute a partitioning of the nodes. This partitioning of nodes is then used to compute a partitioning for the elements. This is done by assigning each element to the partition in which the majority of its nodes belong to (subject to balance constraints).

METIS_PartMeshDual (int *ne, int *nn, idxtype *elmnts, int *etype, int *numflag, int *nparts, int *edgecut, idxtype *epart, idxtype *npart)

Description

This function is used to partition a mesh into *k* equal-size parts. It provides the functionality of the `partdmesh` program.

Parameters

ne The number of elements in the mesh.

nn The number of nodes in the mesh.

elmnts The element node array storing the mesh as described in Section 5.2.

etype Indicates the type of the elements in the mesh. *etype* can take the following values:

1 The elements are triangles.
2 The elements are tetrahedra.
3 The elements are hexahedra (bricks).
4 The elements are quadrilaterals.

numflag Used to indicate which numbering scheme is used for the element node array. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0
1 Fortran-style numbering is assumed that starts from 1

nparts The number of parts to partition the mesh.

edgecut Upon successful completion, this variable stores the number of edges that are cut by the partition in the dual graph.

epart This is a vector of size *ne* that upon successful completion stores the partition vector for the elements of the mesh. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

npart This is a vector of size *nn* that upon successful completion stores the partition vector for the nodes of the mesh. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function converts the mesh into a dual graph and then uses METIS_PartGraphKway to compute a partitioning of the elements. This partitioning of elements is then used to compute a partitioning for the nodes. This is done by assigning each node to the partition in which the majority of its incident elements belong to (subject to balance constraints).

5.6 Sparse Matrix Reordering Routines

METIS_EdgeND (int *n, idxtype *xadj, idxtype *adjncy, int *numflag, int *options, idxtype *perm, idxtype *iperm)

Description

This function computes fill reducing orderings of sparse matrices using the multilevel nested dissection algorithm. It provides the functionality of the `oemetis` program.

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determined the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments has shown that both HEM and SHEM perform quite well.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Region Growing (Default)

options[3] Determines the algorithm used for refinement. Possible values are:

1 Early-Exit Boundary FM refinement (Default)

options[4] Used for debugging purposes. Always set it to 0 (Default).

perm, iperm
These are vectors, each of size n. Upon successful completion, they store the fill-reducing permutation and inverse-permutation. Let A be the original matrix and A' be the permuted matrix. The arrays *perm* and *iperm* are defined as follows. Row (column) i of A' is the perm[i] row (column) of A, and row (column) i of A is the iperm[i] row (column) of A'. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function computes the vertex separator from the edge separator using a minimum cover algorithm. This function should be used only in ordering large graphs arising in 3D finite element applications. In general the METIS_NodeND routine should be preferred, as it produces better orderings.

METIS_NodeND (int *n, idxtype *xadj, idxtype *adjncy, int *numflag, int *options, idxtype *perm, idxtype *iperm)

Description

This function computes fill reducing orderings of sparse matrices using the multilevel nested dissection algorithm. It provides the functionality of the `onmetis` program.

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

options This is an array of 8 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining seven elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments have shown that all three matching schemes perform quite well. In general SHEM is faster and RM is slower, but feel free to experiment with the other matching schemes.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Edge-based region growing (Default)

2 Node-based region growing

options[3] Determines the algorithm used for refinement. Possible values are:

1 Two-sided node FM refinement

2 One-sided node FM refinement (Default)

One-sided FM refinement is faster than two-sided, but in some cases two-sided refinement may produce better orderings. Feel free to experiment with this option.

options[4] Used for debugging purposes. Always set it to 0 (Default).

options[5] Used to select whether or not to compress the graph and to order connected components separately. The possible values and their meaning are as follows.

0 Do not try to compress the graph and do not order each connected component separately.

1 Try to compress the graph. (A compressed graph is actually formed if the size of the graph can be reduced by at least 15%) (Default).

2 Order each connected component of the graph separately. This option is particularly useful when after a few levels of nested dissection, the graph breaks up in many smaller disconnected subgraphs. This is true for certain types of LP matrices.

3 Try to compress the graph and also order each connected component separately.

options[6] Used to control whether or not the ordering algorithm should remove any vertices with high degree (*i.e.*, dense columns). This is particularly helpful for certain classes of LP matrices, in which there a few vertices that are connected to many other vertices. By removing these vertices prior to ordering, the quality and the amount of time required to do the ordering improves. The possible values are as follows:

- 0 Do not remove any vertices (Default)
- x Where $x > 0$, instructs the algorithm to remove any vertices whose degree is greater than $0.1 * x *$ (average degree). For example if $x = 40$, and the average degree is 5, then the algorithm will remove all vertices with degree greater than 20. The vertices that are removed are ordered last (*i.e.*, they are automatically placed in the top-level separator). Good values are often in the range of 60 to 200 (*i.e.*, 6 to 20 times more than the average).

options[7] Used to determine how many separators to find at each step of nested dissection. The larger the number of separators found at each step, the higher the runtime and better the quality is (in general). The default value is 1, unless the graph has been compressed by more than a factor of 2, in which case it becomes 2. Reasonable values are in the range of 1 to 5. For most problems, a value of 5 increases the runtime by a factor of 3.

perm, iperm

These are vectors, each of size n. Upon successful completion, they store the fill-reducing permutation and inverse-permutation. Let A be the original matrix and A' be the permuted matrix. The arrays *perm* and *iperm* are defined as follows. Row (column) i of A' is the perm[i] row (column) of A, and row (column) i of A is the iperm[i] row (column) of A'. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

Note

This function computes the vertex separator directly by using a multilevel algorithm. This function produces high quality orderings and should be preferred over METIS_EdgeND.

METIS_NodeWND (int *n, idxtype *xadj, idxtype *adjncy, idxtype *vwgt, int *numflag, int *options, idxtype *perm, idxtype *iperm)

Description

This function computes fill reducing orderings of sparse matrices using the multilevel nested dissection algorithm. It is similar to METIS_NodeWND but it assumes that the compression has been already performed prior to calling this routine. It is particularly suited for ordering very large matrices in which the compressed matrix is known a priori.

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

vwgt The weight of the vertices.

numflag Used to indicate which numbering scheme is used for the adjacency structure of the graph. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

options This is an array of 5 integers that is used to pass parameters for the various phases of the algorithm. If *options[0]=0* then default values are used. If *options[0]=1*, then the remaining four elements of *options* are interpreted as follows:

options[1] Determines the matching type. Possible values are:

1 Random Matching (RM)

2 Heavy-Edge Matching (HEM)

3 Sorted Heavy-Edge Matching (SHEM) (Default)

Experiments have shown that all three matching schemes perform quite well. In general SHEM is faster and RM is slower, but feel free to experiment with the other matching schemes.

options[2] Determines the algorithm used during initial partitioning. Possible values are:

1 Edge-based region growing (Default)

2 Node-based region growing

options[3] Determines the algorithm used for refinement. Possible values are:

1 Two-sided node FM refinement

2 One-sided node FM refinement (Default)

One-sided FM refinement is faster than two-sided, but in some cases two-sided refinement may produce better orderings. Feel free to experiment with this option.

options[4] Used for debugging purposes. Always set it to 0 (Default).

perm, iperm
These are vectors, each of size n. Upon successful completion, they store the fill-reducing permutation and inverse-permutation. Let A be the original matrix and A' be the permuted matrix. The arrays *perm* and *iperm* are defined as follows. Row (column) i of A' is the perm[i] row (column) of A, and row (column) i of A is the iperm[i] row (column) of A'. The numbering of this vector starts from either 0 or 1, depending on the value of *numflag*.

5.7 Auxiliary Routines

METIS_MeshToNodal (int *ne, int *nn, idxtype *elmnts, int *etype, int *numflag, idxtype *nxadj, idxtype *nadjncy)

Description

This function is used to convert a mesh into a nodal graph, in a format suitable for METISlib. It provides the function of the `mesh2nodal` program.

Parameters

ne The number of elements in the mesh.

nn The number of nodes in the mesh.

elmnts The element node array storing the mesh as described in Section 5.2.

etype Indicates the type of the elements in the mesh. *etype* can take the following values:

1 The elements are triangles.

2 The elements are tetrahedra.

3 The elements are hexahedra (bricks).

4 The elements are quadrilaterals.

numflag Used to indicate which numbering scheme is used for the element node array. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

nxadj, nadjncy

These arrays store the adjacency structure of the nodal graph. The user must provide arrays that are sufficiently large to store the graph. The size of array *nxadj* is *nn+1* where the size of *nadjncy* depends on the type of the mesh. For triangular-element and hexahedra-element meshes, *nadjncy* should be at least $6 * nn$, for quadrilateral-element meshes, *nadjncy* should be at least $4 * nn$, and for tetrahedra-element meshes, *nadjncy* should be at least $15 * nn$.

Note

The nodal graph is defined as the graph in which each vertex of the graph corresponds to a node in the mesh, and two vertices are connected by an edge if the corresponding nodes a connected by an element.

METIS_MeshToDual (int *ne, int *nn, idxtype *elmnts, int *etype, int *numflag, idxtype *dxadj, idxtype *dadjncy)

Description

This function is used to convert a mesh into a dual graph, in a format suitable for METISlib. It provides the function of the `mesh2nodal` program.

Parameters

ne The number of elements in the mesh.

nn The number of nodes in the mesh.

elmnts The element node array storing the mesh as described in Section 5.2.

etype Indicates the type of the elements in the mesh. *etype* can take the following values:

1 The elements are triangles.

2 The elements are tetrahedra.

3 The elements are hexahedra (bricks).

4 The elements are quadrilaterals.

numflag Used to indicate which numbering scheme is used for the element node array. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

dxadj, dadjncy

These arrays store the adjacency structure of the dual graph. The user must provide arrays that are sufficiently large to store the graph. The size of array *dxadj* is *ne+1* where the size of *dadjncy* depends on the type of the mesh. For triangular-element meshes, *dadjncy* should be at least $3 * ne$, for tetrahedra-element and quadrilateral-element meshes, *dadjncy* should be at least $4 * ne$, and for hexahedra-element meshes, *dadjncy* should be at least $6 * ne$.

Note

The dual graph is defined as the graph in which each vertex of the graph corresponds to an element in the mesh, and two vertices are connected by an edge if the corresponding elements share a face.

METIS_EstimateMemory (int *n, idxtype *xadj, int *adjncy, int *numflag, int *optype, int *nbytes)

Description

This function is used to estimate the amount of memory that will be used by METIS. Even though, METIS dynamically allocates the amount of memory that it needs, this function can be useful in determining if the amount of memory in the system is sufficient for METIS.

Parameters

n The number of vertices in the graph.

xadj, adjncy
The adjacency structure of the graph as described in Section 5.1.

numflag Used to indicate which numbering scheme is used for the element node array. *numflag* can take the following two values:

0 C-style numbering is assumed that starts from 0

1 Fortran-style numbering is assumed that starts from 1

optype Indicates the operation for which the memory will be estimated. *optype* can take the following values:

1 Estimates the memory needed for METIS_PartGraphRecursive and METIS_WPartGraphRecursive.

2 Estimates the memory needed for METIS_PartGraphKway and METIS_WPartGraphKway.

3 Estimates the memory needed for METIS_EdgeND.

4 Estimates the memory needed for METIS_NodeND, but it does not take into account memory saved due to compression.

nbytes Upon return, *nbytes* stores an estimate on the number of bytes that METIS requires.

5.8 C and Fortran Support

The various routines in METISlib can be called from either C or Fortran programs. Using C with METISlib is quite straightforward (as METIS is written entirely in C). However, METISlib fully supports Fortran as well. This support comes in three forms.

1. All the scalar arguments in the routines are passed by reference to facilitate Fortran programs.
2. All the routines take a parameter called *numflag* indicating whether or not the numbering of the graph or mesh starts from 0 or 1. In C programs numbering usually starts from 0, whereas in Fortran programs numbering starts from 1.
3. METISlib incorporates alternative names for each of the routines to facilitate linking the library with Fortran programs. In particular, for every function METISlib provides three additional names, one all capital, one all lower case, and one all lower case with '_' appended to it. For example, for METIS_PartGraphKway, METISlib provides METIS_PARTGRAPHKWAY, metis_partgraphkway, and metis_partgraphkway_. These extra names allow the library to be directly linked into Fortran programs on a wide range of architectures including Cray, SGI, and HP. If you still encounter problems linking with the library let us know so we can include appropriate support.

6 System Requirements and Contact Information

The distribution of METIS contains a number of files, that total to over 22,000 lines of code. It is written entirely in ANSI C, and is portable on most Unix systems that have an ANSI C compiler (the GNU C compiler will do). It has been extensively tested on AIX, SunOS, Solaris, IRIX, Linux, HP-UX, BSD, and Unicos. Instructions on how to build and install METIS can be found in the file `INSTALL` of the distribution.

Even though, METIS contains no known bugs, it does not mean that all of its bugs have been found and fixed. If you find any problems, please send email to *metis@cs.umn.edu*, with a brief description of the problem you have found. Also, any future updates to METIS will be made available on WWW at *http://www.cs.umn.edu/˜metis*.

References

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